

Conformal a-anomaly of some non-unitary 6d superconformal theories

Matteo Beccaria^a, Arkady A. Tseytlin^{1b}

^a*Dipartimento di Matematica e Fisica Ennio De Giorgi,
Università del Salento & INFN, Via Arnesano, 73100 Lecce, Italy*

^b*The Blackett Laboratory, Imperial College, London SW7 2AZ, U.K.*

E-mail: matteo.beccaria@le.infn.it, tseytlin@imperial.ac.uk

ABSTRACT: We compute the conformal anomaly a-coefficient for some non-unitary (higher derivative or non-gauge-invariant) 6d conformal fields and their supermultiplets. We use the method based on a connection between 6d determinants on S^6 and 7d determinants on AdS_7 . We find, in particular, that (1,0) supermultiplet containing 4-derivative gauge-invariant conformal vector has precisely the value of a-anomaly as attributed in [arXiv:1506.03807](https://arxiv.org/abs/1506.03807) (on the basis of R-symmetry and gravitational 't Hooft anomaly matching) to the standard (1,0) vector multiplet. We also show that higher derivative (2,0) 6d conformal supergravity coupled to exactly 26 (2,0) tensor multiplets has vanishing a-anomaly (and also vanishing Casimir energy on 5-sphere). This is the 6d counterpart of the known fact of cancellation of the conformal anomaly in the 4d system of $\mathcal{N} = 4$ conformal supergravity coupled to 4 vector $\mathcal{N} = 4$ multiplets. In the case when 5 of tensor multiplets are chosen to be ghost-like and the conformal symmetry is spontaneously broken by a quadratic scalar constraint the resulting IR theory may be identified with (2,0) Poincaré supergravity coupled to 21=26-5 tensor multiplets. The latter theory is known to be special – it is gravitational anomaly free and results upon compactification of 10d type IIB supergravity on $K3$.

¹Also at Lebedev Institute, Moscow

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1 Introduction

There has been some recent interest in conformal a-anomalies of supersymmetric CFTs in $d = 6$ (see, *e.g.*, [1–3] and refs. therein). This motivates revisiting the question about the computation of conformal anomalies for different types of free 6d conformal fields. We shall relax the condition of unitarity as higher-derivative non-unitary CFTs may be of interest, *e.g.*, as formal UV completions of some low-energy models or as 6d counterparts of some (higher spin) theories in AdS₇.

Given a conformal field that can be coupled to the metric in a reparametrization-invariant way, its action on a curved background should be Weyl invariant. The conformal anomaly of a classically Weyl invariant theory in 6d has the following general form [4–6]

$$(4\pi)^3 \langle T \rangle = b_6 = a \mathcal{E}_6 + W_6 + D_6, \quad W_6 = c_1 I_1 + c_2 I_2 + c_3 I_3. \quad (1.1)$$

Here, $\mathcal{E}_6 = -\epsilon_6 \epsilon_6 RRR$ is the Euler density in six dimensions, W_6 is a combination of three independent Weyl invariants built out of the Weyl tensor ($I_3 \sim C\nabla^2 C$, $I_1, I_2 \sim CCC$) and D_6 is a total derivative term (which is scheme-dependent), see [6] for details.

Conformal anomalies for the simplest 6d free conformal fields (scalar, spinor and 2nd rank antisymmetric tensor) were found in [6]. In particular, for the (2,0) tensor multiplet¹

$$(2,0): \quad W_6 = c \mathcal{W}_6, \quad \mathcal{W}_6 = 96I_1 + 24I_2 - 8I_3, \quad c = -\frac{1}{288}, \quad a = -\frac{7}{1152}. \quad (1.2)$$

Let us note that while in the case of (2,0) supersymmetry there should be a single superinvariant containing W_6 combination, *i.e.* there should be a single c -coefficient, in the (1,0) case there should be apparently two. As follows from the results of [6], for conformal anomalies of both tensor and scalar (1,0) multiplets the coefficients c_i satisfy $c_1 = 2c_2 - 6c_3$.

¹We shall follow the notation of [6] in which a-anomaly of a unitary scalar is negative in $d = 6$ as opposed to the standard choice of $a > 0$ in $d = 4$.

The a-coefficient for conformal higher-spin symmetric tensor gauge fields was computed in [7], both directly (from the partition function on S^6) and also using the AdS/CFT inspired relation to massless higher spin partition function in AdS₇ [8] (see also [9, 10]). The latter method was generalized to arbitrary $SO(2, 6)$ representations in [11].

Here we will apply the results of [11] to present the explicit expressions for the a-anomaly coefficient (and also the Casimir energy E_c on S^5) for several types of non-unitary 6d conformal fields including some low-spin supermultiplets as well as the (2,0) conformal supergravity multiplet. In particular,

- i) We will find that the value of the a-anomaly coefficient indirectly attributed in [2] (on the basis of R-symmetry and gravitational 't Hooft anomaly matching) to (1,0) supersymmetric 6d vector multiplet $a_{(1,0) \text{ vector}} = -\frac{251}{210}a_{(2,0) \text{ tensor}}$ corresponds, in fact, to higher-derivative (non-unitary) superconformal vector multiplet $V^{(1,0)}$ with the Lagrangian²

$$\mathcal{L}_{V^{(1,0)}} \sim F_{\mu\nu} \partial^2 F_{\mu\nu} + \bar{\psi} \not{\partial}^3 \psi + \varphi \partial^2 \varphi. \quad (1.3)$$

The higher-derivative model $F_{\mu\nu} \partial^2 F_{\mu\nu}$ having standard vector gauge invariance is the $s = 1$ member of the conformal higher spin family [15] in $d = 6$ with the kinetic operators $\partial^{2s+d-4} = \partial^{2s+2}$. It represent “massless” conformal field which is different from the “massive” (non gauge invariant) one that has 2-derivative kinetic term which is the $s = 1$ member of the family considered in [16] (cf. also [17] and below). One could think of (1.3) as a UV completion of the standard (2-derivative) scale-invariant but not conformally invariant (1,0) Maxwell multiplet in 6d, though the direct relevance of this non-unitary UV theory in the context of IR RG flow of a-anomaly discussed in [2] is unclear. There is also a tentative connection to non-abelian tensor model of [18] containing 3-form field: in 6d the 4-derivative vector is dual to non-dynamical 3-form field (with conformally invariant kinetic term $(C_3^\perp)^2$, see Appendix) for which the contribution to the conformal anomaly comes from the ghost determinants and is thus of “non-unitary” nature (as in Schwinger model or Einstein gravity in 2d).

- ii) We will compute the a-anomaly for the maximally supersymmetric (2,0) 6d conformal supergravity that has a schematic Lagrangian

$$\mathcal{L} \sim C_{\mu\nu\lambda\rho} \partial^2 C_{\mu\nu\lambda\rho} + \psi_\mu \partial^5 \psi_\mu + \dots \quad (1.4)$$

The non-linear action of this theory can be found as a local UV singular part of the induced action of (2,0) tensor multiplet coupled to the conformal supergravity background. As the expression for the W_6 term in the conformal anomaly (1.1) of the (2,0) multiplet takes a particular form in (1.2), the action of the (2,0) conformal supergravity may be interpreted as a supersymmetric extension of \mathcal{W}_6 . We shall observe that

²The non-linear action for non-abelian version of this multiplet was constructed in [12, 13]. It is not superconformally invariant at the quantum level since the gauge coupling β -function does not vanish (and also has chiral gauge anomaly [14]). Here we consider only the free multiplet.

when this theory is coupled to precisely 26 $(2,0)$ tensor multiplets, the total conformal anomaly a -coefficient vanishes. This is the 6d counterpart of the cancellation of 4d conformal anomalies in the system of $\mathcal{N} = 4$ conformal supergravity coupled to four $\mathcal{N} = 4$ vector multiplets [19, 15].³ This cancellation is curious in view of the following observation. Taking 5 of the 26 tensor multiplets to be ghost-like and spontaneously breaking the superconformal symmetry (and dropping higher derivative terms, *i.e.* considering an IR limit) one ends up, following [20], with a theory of the remaining $26 - 5 = 21$ tensor multiplets coupled to the chiral $(2,0)$ 6d Poincaré supergravity. The latter theory is known to be special: it is gravitational anomaly free and results upon compactification of type IIB supergravity on K3 [21, 22].

We shall start in section 2 with a brief review of the conformal fields in 6d and present the general a -anomaly expression derived in [11] using the technical tools of $\text{AdS}_7/\text{CFT}_6$ connection. In section 3 we will consider several unitary and non-unitary superconformal multiplets involving conformal fields of low spin (scalars, spinors, vectors and 2nd rank antisymmetric tensors) and present the results for the corresponding a -anomaly and Casimir energy. We will study, in particular, the non-unitary higher-derivative $(1,0)$ supermultiplet (1.3). In section 4 we will consider the maximal 6d $(2,0)$ conformal supergravity naturally associated with 7d maximal gauged supergravity with AdS_7 vacuum. We will demonstrate that, when coupled to 26 $(2,0)$ tensor multiplets, this theory has vanishing a -anomaly (and vanishing Casimir energy) and discuss some interpretations of this fact. In Appendix A we will compare the S^6 partition functions for various 6d conformal fields and also present the direct 6d derivation of the a -anomaly for the non-unitary conformal vector field with 2-derivative but not gauge-invariant action and its 4-derivative gauge-invariant counterpart.

2 6d conformal fields and a -anomaly from AdS_7

6d conformal fields correspond to $SO(2,6)$ conformal group representations that will be denoted as $(\Delta; \mathbf{h})$ where $\mathbf{h} = (h_1, h_2, h_3)$ are the $SO(6)$ highest weights or Young tableau labels (h_i are all integers or all half-integers with $h_1 \geq h_2 \geq |h_3|$). The dimension $d(\mathbf{h})$ of the $SO(6)$ representation \mathbf{h} is

$$d(\mathbf{h}) = \frac{1}{12}(1 + h_1 - h_2)(1 + h_2 - h_3)(1 + h_2 + h_3)(2 + h_1 - h_3)(2 + h_1 + h_3)(3 + h_1 + h_2).$$

The unitary irreducible representations of $SO(2,6)$ fall into four classes [23, 24]

$$\begin{aligned} (i) \quad & \Delta \geq \Delta = h_1 + 4, \quad h_1 > h_2 \geq |h_3|; & (ii) \quad & \Delta \geq \Delta = h_1 + 3, \quad h_1 = h_2 > |h_3|; \\ (iii) \quad & \Delta \geq \Delta = h_1 + 2, \quad h_1 = h_2 = \pm h_3; & (iv) \quad & \Delta \geq 2 \text{ or } \Delta = 0, \quad h_1 = h_2 = h_3 = 0. \end{aligned} \tag{2.1}$$

³While we will compute the a -coefficient just for the free field multiplet, *i.e.* the 1-loop contribution, in the maximally supersymmetric case it is likely to be exact (as in the 4d case).

Generic representations are non-degenerate (or “massive”), while representations at the unitarity bounds are maximally degenerate (they correspond, in particular, to conformal higher spins associated with massless higher spin fields in AdS_7 [23]).⁴ There are also intermediate cases of non-unitary conformal fields of non-maximal depth related to partially massless fields in AdS_7 (see [25–27] for general discussions).⁵

Assuming a free conformal field φ with a local field theory description with a free action $S = \int d^d x \varphi \partial^n \varphi$, its canonical dimension should be $\Delta = \frac{d-n}{2}$. For example, a conformal higher spin field represented by a symmetric rank s tensor (i.e. $\mathbf{h} = (s, 0, 0)$) with action that has maximal gauge invariance has $n = 2s + d - 4$, i.e. has canonical $\Delta = 2 - s$ and thus is unitary in $d = 6$ only for $s = 0$ (cf. (2.1)).

To find the corresponding conformal anomaly, one may couple φ to a background metric $g_{\mu\nu}$ (getting a classically Weyl-invariant action with φ transforming with an appropriate Weyl weight) and compute the trace of the variation of the 1-loop effective action $\Gamma = -\log Z[g]$ over $g_{\mu\nu}$. Equivalently, to extract the a -coefficient it is sufficient to find the logarithmic UV divergent part of Γ computed on 6-sphere

$$\Gamma = -B_6 \log(r\Lambda) + \dots, \quad B_6 = \frac{1}{(4\pi)^3} \Omega(S^6) b_6 = \frac{1}{60} b_6 = -96a. \quad (2.2)$$

Here Λ is a UV cutoff and r is the radius of S^6 , see [7] for details.

This direct 6d computation of the a -coefficient appears to require a case-by-case analysis, but there exist a remarkably universal method of computing a -anomaly using the AdS/CFT motivated relation between the determinants of a 2nd order operator in AdS_{d+1} space and of the associated conformal field operator on d -dimensional boundary.⁶ While being a just a technical device (leading to the same results as the 6d computation, as one can check on particular examples), this method makes full use of the underlying conformal symmetry and allows one to compute the a -coefficient for a generic representation $(\Delta; h_1, h_2, h_3)$.

A conformal field φ in \mathbb{R}^d of canonical dimension Δ_- may be interpreted as a shadow (or source) field associated to another conformal field J (or *current*) of dimension $\Delta_+ = d - \Delta_-$ that has the same $SO(d)$ representation labels \mathbf{h} . For example, in the context of vectorial AdS/CFT , the current J may be interpreted as a bilinear in complex scalars which is dual to a massless higher spin field ϕ in AdS_{d+1} transforming in the same representation of $SO(2, d)$ (the isometry group of AdS_{d+1}) as J , i.e. $(\Delta_+; \mathbf{h})$.

⁴ Their characters can be written by suitable subtractions in terms of the massive (generic) representation character $\widehat{\mathcal{Z}}(\Delta; h_1, h_2, h_3) = d(\mathbf{h}) \frac{q^\Delta}{(1-q)^6}$. For example, in the $\Delta = h_1 + 4$ case (i), we have the following massless character (see [11] for details) $\mathcal{Z}(h_1 + 4; h_1, h_2, h_3) = \widehat{\mathcal{Z}}(h_1 + 4; h_1, h_2, h_3) - \widehat{\mathcal{Z}}(h_1 + 5; h_1 - 1, h_2, h_3)$.

⁵ In general, the origin of non-unitarity of a free conformal theory may be due to higher derivative kinetic term (as in conformal higher spin field case) and/or reduced gauge invariance that does not allow to eliminate all ghost-like components.

⁶ This relation has a kinematic origin and belongs to a general class of bulk-boundary relations discussed in [28, 29]. Its AdS/CFT interpretation involves the bulk counterpart of a “double trace” deformation of the boundary CFT (see, in particular, [30–32] for scalar operators). In [8, 7, 9] it was applied to the computation of the a -coefficient of totally-symmetric higher-spin conformal fields and in [10, 11] it was generalized to arbitrary conformal representations in 4d and 6d (see also [33, 34, 10] for related general discussions).

For a generic field ϕ , the associated current needs not be conserved and the dual AdS_{d+1} field ϕ is massive, *i.e.* its action $\sim \int d^{d+1}x \sqrt{g} \phi(-\nabla^2 + m^2)\phi$ has no gauge invariance. Considering the ratio of determinants of the kinetic operator of ϕ with Dirichlet (+) and Neumann (-) boundary conditions one can then argue (cf. [28, 30–33]) that their ratio should be related to the determinant of the kinetic operator of the boundary conformal field ϕ , *i.e.* the corresponding 1-loop partition functions should be related as

$$Z_\phi(\mathcal{M}^d) = \frac{Z_\phi^-(\text{AdS}_{d+1})}{Z_\phi^+(\text{AdS}_{d+1})}, \quad (2.3)$$

where \mathcal{M}^d is the boundary of AdS_{d+1} (*e.g.*, S^d). The same relation applies also in the *reducible* cases with gauge invariance, *e.g.* the partition function of a conformal higher spin field ϕ_s is related to the ratio of the \pm partition functions of the AdS_{d+1} massless higher spin field ϕ_s . In the even d case $Z_\phi(\text{AdS}_{d+1})$ is UV finite but logarithmically IR divergent (due to the AdS volume factor), while $Z_\phi(\mathcal{M}^d)$ is IR finite but logarithmically UV divergent as in (2.2). Identifying the two cutoffs allows one to find the a -coefficient for ϕ by computing the field ϕ determinants in AdS_{d+1} [8, 7, 9–11]. As a result, the a -coefficient for ϕ is $a = a^+ - a^-$ where $a^+ = f(\Delta_+, \mathbf{h})$, $a^- = f(\Delta_-, \mathbf{h})$. As it turns out, $f(\Delta) = -f(d - \Delta)$, *i.e.* $a^- = -a^+$, so that

$$a(\Delta; \mathbf{h}) = -2a^+(\Delta, \mathbf{h}), \quad \Delta \equiv \Delta_+ = d - \Delta_-. \quad (2.4)$$

In what follows we shall follow [10, 11] and label the conformal field representation not by its canonical dimension Δ_- but by the dimension $\Delta_+ = d - \Delta_-$ of the dual AdS field. The corresponding a -coefficients for fields of dimension Δ and $d - \Delta$ differ only by the overall sign, see (2.5) below where Δ will also stand for Δ_+ (the discussion of (non)unitarity of a given conformal field should of course be based on its canonical dimension Δ_-).

In particular, in the $d = 6$ case one finds for a generic massive $SO(2, 6)$ representation [11] ($\mathbf{h} = (h_1, h_2, h_3)$, $\bar{h} = h_1 + h_2 + h_3$)

$$\begin{aligned} a(\Delta; \mathbf{h}) = & -\frac{(-1)^{2\bar{h}} d(\mathbf{h})}{96 \times 37800} (\Delta - 3) \left[15(\Delta - 3)^6 \right. \\ & - 21(\Delta - 3)^4 [h_3^2 + h_1(h_1 + 4) + h_2(h_2 + 2) + 5] \\ & + 35(\Delta - 3)^2 [(h_1 + 2)^2 (h_2 + 1)^2 + (h_1(h_1 + 4) + h_2(h_2 + 2) + 5)h_3^2] \\ & \left. - 105(h_1 + 2)^2 (h_2 + 1)^2 h_3^2 \right]. \end{aligned} \quad (2.5)$$

In the case of degenerate representations (*e.g.*, short ones saturating a unitarity bound), one needs to combine the corresponding massive representation expressions appropriately (*i.e.* subtract “ghost” contributions).

One may also apply a similar method to compute the Casimir energy on S^{d-1} by using (2.3) in the case of the boundary being $\mathbb{R} \times S^{d-1}$ [35, 34, 10]. The general expression for the Casimir energy on S^5 of a 6d conformal field is found to be [11]

$$E_c(\Delta; \mathbf{h}) = -\frac{(-1)^{2\bar{h}} d(\mathbf{h})}{60480} (\Delta - 3) \left[12(\Delta - 3)^6 - 126(\Delta - 3)^4 + 336(\Delta - 3)^2 - 191 \right]. \quad (2.6)$$

Note that the expression for E_c is (in contrast to the one for a in (2.5)) scheme dependent in general: it is determined by the a -coefficient but also by the scheme-dependent coefficients of the total derivative terms in D_6 in (1.1) [36]. Here we use a particular (heat kernel or ζ -function) scheme in which a and E_c are not simply proportional (cf. [37]) so that the computation of E_c provides an independent information about the D_6 terms in (1.1).

3 Unitary and non-unitary low spin 6d conformal multiplets

Let us consider generic (higher derivative) 6d conformal theories for a scalar (φ), spin $\frac{1}{2}$ fermion (ψ), vector (V), and 2nd rank antisymmetric tensor (T). We adopt a notation that displays the order of derivatives in the corresponding kinetic term

$$\varphi^{(n)} \rightarrow \varphi \partial^n \varphi, \quad \psi^{(n)} \rightarrow \psi \not{\partial}^n \psi, \quad V^{(n)} \rightarrow V_\mu \partial^n V_\mu, \quad T^{(n)} \rightarrow T_{\mu\nu} \partial^n T_{\mu\nu}. \quad (3.1)$$

In this section we shall focus on particular cases $\varphi \equiv \varphi^{(2)}, \varphi^{(4)}$; $\psi \equiv \psi^{(1)}, \psi^{(3)}$; $V^{(2)}, V^{(4)}$, and the standard $T \equiv T^{(2)}$ with self-dual field strength H .⁷

In $d = 6$ the conformal field $V^{(2)}$ has canonical dimension $\Delta_- = 2$ (i.e. the corresponding dual representation is $(4; 1, 0, 0)$) and thus is below the unitarity bound in (2.1). Indeed, it is not described by the usual Maxwell Lagrangian $\sim F_{\mu\nu} F_{\mu\nu}$ but rather by a non gauge-invariant one $(\partial_\mu V_\nu^{(2)})^2 - \frac{3}{2}(\partial_\mu V_\mu^{(2)})^2$ (see Appendix A) which is a special $s = 1$ case of a class of 2nd derivative conformal spin s field actions discussed in [16]. Indeed, the Maxwell vector is scale invariant but not conformally invariant in $d = 6$ (see, e.g., [17]).

At the same time, the field $V^{(4)}$ is described by the gauge invariant conformal theory with the kinetic term $\sim F_{\mu\nu} \partial^2 F_{\mu\nu}$ where $F_{\mu\nu} = \partial_\mu V_\nu^{(4)} - \partial_\nu V_\mu^{(4)}$. This is the $s = 1$ member of the conformal higher spin family in $d = 6$ [15, 33, 7]. Its canonical dimension $\Delta_- = 1$, i.e. it is also non-unitary (cf. (2.1)) but now the non-unitarity may be attributed to its higher-derivative kinetic term. The corresponding Weyl-invariant action on curved background is presented in Appendix (see (A.8)).

The count of dynamical (on-shell) degrees of freedom ν for these fields goes as follows. Since for the 2-derivative scalar $\nu(\varphi) = 1$ we have $\nu(\varphi^{(n)}) = n/2$. For 6d Majorana-Weyl fermion $\psi^{(n)}$ we get $\nu(\psi^{(n)}) = -2n$. The standard gauge-invariant antisymmetric tensor $T_{\mu\nu}$ with selfdual field strength has $\nu(T) = 3$. The conformal vector $V^{(n)}$ with $n \neq 4$ has no gauge invariance so $\nu(V^{(n \neq 4)}) = d \times n/2 = 3n$. The case $n = 4$ in 6d is special: because of gauge invariance the action is $V_\mu^\perp \square^2 V_\mu^\perp$ and there is also a factor $(\det \square)^{1/2}$ coming from the measure (after one sets $V_\mu = V_\mu^\perp + \partial_\mu \sigma$ and divides over the gauge group volume). As a result, $\nu(V^{(4)}) = (6 - 1) \times 2 - 1 = 9$.

Applying (2.5), (2.6) one can compute the a -anomaly and the Casimir energy corresponding to these conformal fields. Table 1 lists the (dual or AdS-adapted) representation content, on-shell degrees of freedom, a -anomaly, and Casimir energy of these 6d conformal fields. Note that the value for the a -anomaly for $V^{(4)}$ is indeed the same as for $s = 1$ conformal higher spin field in 6d found in [7], see also Appendix.

⁷As usual, when talking about actions for self-dual antisymmetric tensor fields we will be assuming that self-duality condition is relaxed and imposed on equations of motion or in relevant quantum computation.

field	$SO(2,6)$	ν	$7! \text{ a}$	$7! E_c$
φ	$(4; 0, 0, 0)$	1	$-\frac{5}{72}$	$-\frac{31}{12}$
$\varphi^{(4)}$	$(5; 0, 0, 0)$	2	$\frac{4}{9}$	$\frac{95}{6}$
ψ	$(\frac{7}{2}; \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$	-2	$-\frac{191}{288}$	$-\frac{1835}{96}$
$\psi^{(3)}$	$(\frac{9}{2}; \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$	-6	$\frac{39}{32}$	$\frac{1021}{32}$
$V^{(2)}$	$(4; 1, 0, 0)$	6	-1	$-\frac{31}{2}$
$V^{(4)}$	$(5; 1, 0, 0) - (6; 0, 0, 0)$	9	$\frac{275}{8}$	$\frac{1755}{4}$
T	$\frac{1}{2}[(4; 1, 1, 0) - (5; 1, 0, 0) + (6; 0, 0, 0)]$	3	$-\frac{221}{8}$	$-\frac{955}{4}$

Table 1. Some 6d conformal fields and their properties. Here $SO(2,6)$ quantum numbers $(\Delta; h_1, h_2, h_3)$ refer to the dual field, *i.e.* $\Delta = 6 - \Delta_-$ where Δ_- is canonical dimension of a conformal field. Listed are the numbers of dynamical d.o.f. ν and the values of the a-anomaly and the Casimir energy E_c on S^5 . Combinations of representations account for shortening (gauge freedom). Here T has self-dual field strength (which is indicated by $1/2$ in representation content).

These fields may be combined into supermultiplets with zero total number of degrees of freedom $\nu = 0$. In particular, we may consider $(1,0)$ hyper, tensor and vector superconformal multiplets

$$S^{(1,0)} = 4\varphi + 2\psi, \quad T^{(1,0)} = \varphi + 2\psi + T, \quad V^{(1,0)} = 3\varphi + 2\psi^{(3)} + V^{(4)}, \quad (3.2)$$

as well as the $(2,0)$ tensor multiplet

$$T^{(2,0)} = T^{(1,0)} + S^{(1,0)} = 5\varphi + 4\psi + T. \quad (3.3)$$

While the unitary $S^{(1,0)}$, $T^{(1,0)}$ and $T^{(2,0)}$ are familiar, the non-unitary multiplet $V^{(1,0)}$ may be less so. Its Lagrangian (1.3) is essentially like the standard Maxwell supermultiplet Lagrangian with an extra ∂^2 operator in the kinetic terms (see [12]).

Expressing the a-anomaly and the Casimir energy for these multiplets in terms of their values for the $(2,0)$ tensor multiplet found in [6, 11] we obtain

$$\begin{aligned} \text{a}(S^{(1,0)}, T^{(1,0)}, V^{(1,0)}) &= \left(\frac{11}{210}, \frac{199}{210}, -\frac{251}{210} \right) \text{a}(T^{(2,0)}), \\ E_c(S^{(1,0)}, T^{(1,0)}, V^{(1,0)}) &= \left(\frac{37}{250}, \frac{213}{250}, -\frac{377}{250} \right) E_c(T^{(2,0)}), \\ \text{a}(T^{(2,0)}) &= -\frac{7}{1152}, \quad E_c(T^{(2,0)}) = -\frac{25}{384}. \end{aligned} \quad (3.4)$$

We observe that the value $-\frac{251}{210} \text{a}(T^{(2,0)})$ attributed (on the basis of 't Hooft anomaly matching) in [2] to the standard (non-conformal) $(1,0)$ Maxwell multiplet corresponds, in fact, to the non-unitary higher-derivative $V^{(1,0)}$ multiplet.

Let us make few comments to try to understand this coincidence (see also a discussion of non-unitary $V^{(4)}$ field and its 3-form dual in Appendix). An important point should

be that non-conformal Maxwell vector multiplet should be emerging upon spontaneous breaking of conformal invariance from a conformal system of interacting vector and tensor multiplets with $\varphi F_{\mu\nu} F^{\mu\nu}$ term in the Lagrangian [18] where φ is a scalar of tensor multiplet that has dimension 2 in 6d. That means this vector has canonical dimension 1 as for 4-derivative vector field and that changes also the assignment of R-charge to the corresponding fermion and then the count of anomalies and thus of a-anomaly should go as in $V^{(1,0)}$ multiplet case. Starting with non-conformal Maxwell (1,0) 6d multiplet with symbolic Lagrangian $m^2(F_{\mu\nu}F^{\mu\nu} + \psi\partial\psi + D^2)$ (where $m^2 = < \varphi >$) one may embed it into a higher-derivative theory $F_{\mu\nu}(m^2 + \partial^2)F^{\mu\nu} + \psi(m^2 + \partial^2)\partial\psi + D(m^2 + \partial^2)D$ which is conformal in the UV. The axial anomalies will be the same (as ∂ and ∂^3 fermions have the same chiral anomalies, see, e.g., [14]) but a-anomaly will be controlled by the conformal UV theory, i.e. will be the same as for the $V^{(1,0)}$ multiplet in (3.2).

A similar example (suggested in [2]) exists in 4 dimensions: starting with a non-conformal linear multiplet $(B_{\mu\nu}, \varphi, \psi, f)$ one can dualize the antisymmetric tensor⁸ into a scalar getting a non-conformal analog of the chiral multiplet (with the dual scalar shift symmetry prohibiting $R\varphi^2$ term on a curved background) where the $U(1)_R$ charge of the scalar should be 0 and that of the associated fermion being -1. If one then formally applies the relation between the R-charge and a-anomaly coefficient which is valid in superconformal $\mathcal{N} = 1$ theories one finds that the associated conformal anomaly coefficient should be $a_{d=4} = -\frac{3}{16}$ [2]. In view of the above discussion we should expect that this value should actually correspond to the conformally invariant higher-derivative version [40] of the chiral multiplet with extra ∂^2 in kinetic terms, i.e. with the action $\int d^4x d^4\theta \Phi \partial^2 \Phi \rightarrow \int d^4x (\varphi^* \partial^4 \varphi + \psi \partial^3 \psi + f^* \partial^2 f)$. Adding extra derivatives effectively shifts the scaling dimension and R-charge assignments compared to the standard chiral multiplet ones. Indeed, starting with its Weyl-invariant generalization to curved space and computing the corresponding a-anomaly contributions [41, 15] (see, e.g., Table 2 in [10]) one finds⁹

$$a_{d=4} = 2 \times \left(-\frac{7}{90}\right) - \frac{3}{80} + 2 \times \frac{1}{360} = -\frac{3}{16}. \quad (3.5)$$

Going back to the 6d values of a-anomaly in (3.4), for a combination $n_s S^{(1,0)} + n_t T^{(1,0)} + n_v V^{(1,0)}$ of several multiplets we get

$$\begin{aligned} a(n_s, n_t, n_v) &= \frac{1}{210} (11n_s + 199n_t - 251n_v) a(T^{(2,0)}), \\ E_c(n_s, n_t, n_v) &= \frac{1}{250} (37n_s + 213n_t - 377n_v) E_c(T^{(2,0)}). \end{aligned} \quad (3.6)$$

⁸4d theory of gauge-invariant antisymmetric tensor $H_{\mu\nu\lambda} H^{\mu\nu\lambda}$ is not conformal already at the classical level, so the notion of conformal anomaly does not apply. One may still formally consider the corresponding B_4 coefficient of logarithmic UV divergence of partition function on a curved background (e.g., on S^4) and thus define the “analog” of the one-loop a-coefficient (cf. (2.2)); its value will be preserved by the duality transformation in the path integral (see, e.g., [38, 39]).

⁹The same result is found by using the AdS₅ motivated count of a-anomaly based on the analog [10] of eq.(2.5), i.e. summing up the contributions of the corresponding $SO(2,4)$ representations: $a_{d=4} = 2a(4;0,0) + [a(\frac{7}{2};\frac{1}{2},0) + a(\frac{7}{2};0,\frac{1}{2})] + 2a(3;0,0)$.

Since the non-unitary vector multiplet gives a negative contribution as compared to the two unitary ones it is of interest to see if the a-anomaly cancellation condition $a(n_s, n_t, n_v) = 0$ has simple solutions for integers n_s, n_t, n_v . We find (here q_i are non-negative integers)

$$\begin{aligned} n_s &= 251q_1 + 96q_2 + 37q_3 + 15q_4 + 8q_5 + q_6, & n_t &= q_2 + 3q_3 + 8q_4 + 21q_5 + 34q_6 + 251q_7, \\ n_v &= 11q_1 + 5q_2 + 4q_3 + 7q_4 + 17q_5 + 27q_6 + 199q_7. \end{aligned}$$

Two special cases are $(n_s, n_t, n_v) = (15, 8, 7)$ and $(8, 21, 17)$.¹⁰

4 (2,0) Conformal Supergravity in 6d

Let us now consider an example of higher spin 6d supermultiplet – maximally extended (2,0) conformal supergravity (CSG). While as a background (off-shell) multiplet coupled to (2,0) tensor multiplet it was constructed earlier in [20], the corresponding action for dynamical CSG fields was not discussed in the past.

The strategy to determine this action may be the same as in 4d case where the action of $\mathcal{N} = 4$ CSG can be found as an *induced* one: either as a local UV singular part of the one-loop effective action of $\mathcal{N} = 4$ Maxwell multiplet coupled to $\mathcal{N} = 4$ CSG background or as an IR singular part of the value of the $\mathcal{N} = 8, d = 5$ gauged supergravity action evaluated on solution of the corresponding Dirichlet problem [42, 43]. Similarly, in the $d = 6$ case we may consider either the UV divergent term in the induced action for (2,0) tensor multiplet coupled to (2,0) CSG multiplet or the IR singular part [44] of the value of the action of maximal 7d gauged supergravity [45, 46]. In particular, the structure of the conformal anomaly for (2,0) tensor multiplet [6] implies that the CSG action (1.4) should be the supersymmetric extension of the corresponding special \mathcal{W}_6 term in (1.2).¹¹ This definition of the CSG action as the local part of the induced theory guarantees the right symmetries and thus allows in principle to find its full non-linear form.

¹⁰ If we require in addition $E_c(n_s, n_t, n_v) = 0$, then the most general solution is $n_s = 1078q_1$, $n_t = 257q_1$, $n_v = 251q_1$, where q_1 is a non-negative integer.

¹¹ This special Weyl invariant was first suggested as a “simplest one” in [4]. It can be written only in terms of Ricci tensor and that is why it came out also in the holographic computation [44] of the c-part (i.e. W_6 in (1.1)) of the conformal anomaly of strongly coupled (2,0) theory (i.e. as the logarithmic IR divergence of the 7d Einstein action evaluated on solution with prescribed boundary metric). Being protected by maximal 6d supersymmetry this term appears [6] also in the conformal anomaly of free (2,0) tensor multiplet (1.2). The fact that the Weyl invariant \mathcal{W}_6 has a particular structure (it can be expressed in terms of Ricci tensor and is at most linear in Weyl tensor if one drops a total derivative term $\sim \mathcal{E}_6$) implies that it can be rewritten as a 2nd derivative action involving several tensors of rank ≤ 2 and it is uniquely selected by this requirement [47]. It is also a natural choice for the Weyl gravity action in 6 dimensions which shares the 4d property that its classical “S-matrix” for the physical graviton mode in dS_6 (or euclidean AdS_6) is the same as in the Einstein theory [48] (as this combination \mathcal{W}_6 appears also in the corresponding regularized 6d volume in [49]).

field		$SO(2,6)$	$USp(4)$ dim	$7! a$	$7! E_c$
scalar	$D^{ij,k\ell}$	$(4; 0, 0, 0)$	$[0, 2]$ 14	$-\frac{5}{72}$	$-\frac{31}{12}$
fermion	χ_k^{ij}	$(\frac{9}{2}; \frac{1}{2}, \frac{1}{2}, -\frac{1}{2})$	$[1, 1]$ 16	$\frac{39}{32}$	$\frac{1021}{32}$
3-form	$T_{\mu\nu\lambda}^{ij}$	$(5; 1, 1, -1)$	$[0, 1]$ 5	$\frac{166}{9}$	$\frac{475}{3}$
vector	V_μ^{ij}	$(5; 1, 0, 0) - (6; 0, 0, 0)$	$[2, 0]$ 10	$\frac{275}{8}$	$\frac{1755}{4}$
gravitino	ψ_μ^i	$(\frac{11}{2}; \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}) - (\frac{13}{2}; \frac{1}{2}, \frac{1}{2}, -\frac{1}{2})$	$[1, 0]$ 4	$-\frac{4643}{16}$	$-\frac{137637}{16}$
graviton	$h_{\mu\nu}$	$(6; 2, 0, 0) - (7; 1, 0, 0)$	$[0, 0]$ 1	$\frac{3005}{2}$	37287

Table 2. Representations, a-anomaly and Casimir energy for fields of (2,0) 6d conformal supergravity. As in Table 1 the $SO(2,6)$ representations $(\Delta; h_1, h_2, h_3)$ refer to dual fields of AdS_7 supergravity, *i.e.* $\Delta = 6 - \Delta_-$ where Δ_- is canonical dimension of the 6d conformal field.

To compute the associated a-anomaly coefficient and Casimir energy we may use again the general expressions (2.5) and (2.6). This was essentially done already in [11]. The relevant field representation content is readily determined, using, *e.g.*, the relation to the maximal gauged 7d supergravity (with AdS_7 vacuum) which is the bottom level of the Kaluza-Klein tower of multiplets corresponding to 11d supergravity compactified on S^4 (for translation between fields of 7d gauged and 6d conformal supergravities see [20, 50]). The resulting data is presented in Table 2.¹²

The number of derivatives in kinetic terms is determined by the dimensions of the fields. $D^{ij,k\ell}$ is a conformal scalar with canonical dimension $\Delta_- = 6 - 4 = 2$, *i.e.* is the standard scalar φ discussed in Sec. 3. χ_k^{ij} is a conformal spin $\frac{1}{2}$ fermion spinor with $\Delta_- = \frac{3}{2}$, *i.e.* it is the 3-derivative fermion $\psi^{(3)}$ in (3.1). $T_{\mu\nu\lambda}^{ij}$ is a non gauge invariant (anti) selfdual antisymmetric 3rd rank tensor with $\Delta_- = 1$, *i.e.* it should have kinetic term $T\partial^4 T$.

The remaining three fields in Table 2 have maximal gauge invariance, *i.e.* they are members of conformal higher spin family. The conformal vector V_μ^{ij} with $\Delta_- = 1$ is of $V^{(4)}$ type in (3.1). The 4 conformal gravitini¹³ ψ_μ^i have $\Delta_- = \frac{1}{2}$ and thus the Lagrangian $\sim \psi_\mu \partial^5 \psi_\mu$. The graviton has $\Delta_- = 0$, *i.e.* its kinetic term has 6 derivatives as appropriate for the conformal gravity in 6d ($L \sim C \partial^2 C + \dots$, see, *e.g.*, [47]).

To summarize, the symbolic form of the linearized Lagrangian of the maximal (2,0) 6d conformal supergravity is (here $\psi_{\mu\nu}$ and $F_{\mu\nu}$ are the gravitino and the vector field

¹²Here $i, j, \dots = 1, \dots, 4$ are $USp(4)$ indices and μ, ν, \dots are 6d indices. Let us note that the same values of the Casimir energy for these fields were previously found in [51] (see also [11]).

¹³We consider all 6d spinors as Majorana-Weyl, *i.e.* there are 2 Weyl gravitini corresponding to (2, 0) supersymmetry.

strengths)¹⁴

$$L_{(2,0) \text{ CSG}} \sim C_{\mu\nu\lambda\rho} \partial^2 C_{\mu\nu\lambda\rho} + \psi_{\mu\nu}^i \partial^3 \psi_{\mu\nu}^i + F_{\mu\nu}^{ij} \partial^2 F_{\mu\nu}^{ij} + T_{\mu\nu\lambda}^{ij} \partial^4 T_{\mu\nu\lambda}^{ij} + \chi_k^{ij} \partial^3 \chi_k^{ij} + D^{ij,kl} \partial^2 D^{ij,kl} . \quad (4.1)$$

This resembles the Lagrangian of $\mathcal{N} = 4$ CSG in 4d [52, 15] but with extra ∂^2 factors in the kinetic terms (suggesting a relation via some sort of dimensional reduction). In full non-linear theory the vector V_μ^{ij} will be a $USp(4) = SO(5)$ non-abelian gauge field gauging the R-symmetry of the superconformal group (i.e. coupled to the R-symmetry current of the (2,0) tensor multiplet).

The total values of the a-anomaly and E_c for the (2,0) conformal supergravity multiplet are obtained by summing the contributions of all the fields in Table 2 taking into account their $USp(4)$ multiplicities (cf. (3.4))

$$\begin{aligned} a(\text{CSG}^{(2,0)}) &= \frac{91}{576} = -26 a(T^{(2,0)}) , \\ E_c(\text{CSG}^{(2,0)}) &= \frac{325}{192} = -26 E_c(T^{(2,0)}) . \end{aligned} \quad (4.2)$$

Remarkably, these values mean that (2,0) conformal supergravity coupled to 26 (2,0) tensor multiplets has vanishing a-anomaly and the Casimir energy. The fact that the values of a-anomaly and E_c are correlated implies the cancellation of the total derivative D_6 term in (1.1) in this maximally supersymmetric case.¹⁵

This cancellation is the 6d counterpart of the vanishing of anomalies (and Casimir energy) in the system of $\mathcal{N} = 4$ CSG plus 4 $\mathcal{N} = 4$ vector multiplets in 4d [15] where the analog of (4.2) is

$$(a, c, E_c)(\text{CSG}^{\mathcal{N}=4}) = -4 (a, c, E_c)(V^{\mathcal{N}=4}) . \quad (4.3)$$

By analogy with the 4d system where both a- and c-anomaly coefficients cancel it is natural to conjecture also the cancellation of the Weyl-invariant W_6 part of the 6d conformal anomaly in (1.1). This suggests that¹⁶

$$c(\text{CSG}^{(2,0)}) = -26 c(T^{(2,0)}) = \frac{13}{144} . \quad (4.4)$$

¹⁴The check that the total number of dynamical d.o.f. vanishes goes as follows. T obeys self-dual condition (the sign of its $SO(6)$ label h_3 is fixed) giving factor $\frac{1}{2}$; it has kinetic term $\sim \square^2$ and $\frac{6 \cdot 5 \cdot 4}{3!}$ components with no gauge invariance, therefore $\nu(T) = 20$. For the 6d conformal graviton and gravitino, we can generalize the counting in [33]. For a bosonic conformal higher spin field with spin s in d dimensions we have $\nu_s = (s + \frac{d-4}{2}) N_s - (s + 1 + \frac{d-4}{2}) N_{s-1}$, with $N_s = \binom{s+d-1}{s} - \binom{s+d-3}{s-2}$. For half-integer $s = s + \frac{1}{2}$, we may use the same expression, but with $N_s = q [\binom{s+d-1}{s} - \binom{s+d-2}{s-1}]$, where $q = \frac{1}{2} 2^{d/2}$ for Majorana-Weyl spinors. This gives $\nu(h_{\mu\nu}) = 36$ and $\nu(\psi_\mu) = -36$. Thus finally

$$\nu(\text{CSG}^{(2,0)}) = 14 \times 1 + 16 \times (-6) + 5 \times 20 + 10 \times 9 + 4 \times (-36) + 36 = 0 .$$

¹⁵Let us mention that one may also consider the case of less supersymmetric (1,0) 6d CSG [53] whose (non-auxiliary) field content is the same as in Table 2, but with multiplicities $D(1), \chi(2), T(1), V_\mu(3), \psi_\mu(2), h_{\mu\nu}(1)$, so that the total number of dynamical d.o.f. is again zero:

$$\nu(\text{CSG}^{(1,0)}) = 1 \times 1 + 2 \times (-6) + 1 \times 20 + 3 \times 9 + 2 \times (-36) + 36 = 0 .$$

One finds that in this case: $a(\text{CSG}^{(1,0)}) = \frac{797}{3840}$, $E_c(\text{CSG}^{(1,0)}) = \frac{16471}{3840}$. It is possible to arrange the total a-anomaly to vanish by adding (1,0) matter multiplets (cf. (3.6)) in many ways, none of which seems particularly special.

¹⁶One expects that the maximal (2,0) supersymmetry implies the appearance of a unique Weyl invariant in the conformal anomaly (1.1) – the combination \mathcal{W}_6 in (1.2) which was found in the free tensor multiplet

In this case the system of (2,0) CSG coupled to 26 tensor multiplets will be completely anomaly-free¹⁷ (in particular, UV finite) and thus formally consistent as a quantum theory.

It is interesting to note here a possible connection¹⁸ to eq. (3.11) in [57] $c_{(2,0)}/c(T^{(2,0)}) = c_{2d}$ which relates the ratio of c-coefficients of a given (2,0) CFT and of the (2,0) tensor multiplet to the central charge of some associated 2d chiral algebra. If (4.4) is true, then in the present case of (2,0) CSG this ratio should be $c_{2d} = -26$. It is then natural to interpret this as central charge or conformal anomaly of pure 2d Einstein gravity (which has trivial action and thus is classically Weyl invariant): its anomaly comes just from the ghost determinant contributing the famous -26 (which can be cancelled by adding 26 scalar fields as in the bosonic string) [58].

Remarkably, a similar relation exists for the conformal anomaly c-coefficient of 4d conformal supergravity. According to [59] in the case of $\mathcal{N} = 2$ 4d superconformal theories one should have for the associated 2d central charge $c_{2d} = -12c_{4d}$ where c_{4d} is the 4d conformal anomaly c-coefficient (normalised to be 1/6 for a vector multiplet). This gives $c_{2d} = -12 \times \frac{13}{6} = -26$ in the case of $\mathcal{N} = 2$ CSG [41], which should thus be also associated to pure 2d gravity.

To draw a further parallel between the above 4d and 6d results let us recall the relation of (4.2) and (4.3) to quantum properties of KK supermultiplets appearing in compactification of 11d supergravity on $\text{AdS}_7 \times S^4$ [11] and 10d supergravity on $\text{AdS}_5 \times S^5$ [10] respectively. In both cases each level of the KK tower is labeled by an integer $p \geq 1$ and contains a set $\Phi^{(p)}$ of AdS_{d+1} fields with definite quantum numbers. The fields at level $p = 1$ represent a (formally decoupled) singleton – (2,0) 6d tensor multiplet $T^{(2,0)}$ in the former case and $\mathcal{N} = 4$ 4d vector multiplet $V^{\mathcal{N}=4}$ in the latter case. The maximal gauged 7d or 5d supergravity fields are found at level $p = 2$, while $p > 2$ levels contain massive states. Eq. (3.4) of [11] and eq. (6.7) of [10] give contributions to the a-anomaly (proportional to the coefficient in the 1-loop vacuum partition function in AdS_{d+1}) of the $\Phi^{(p)}$ supermultiplets at level p in 6d and 4d cases respectively:

$$6d : a(\Phi^{(p)}) = -2(6p^2 - 6p + 1)a(T^{(2,0)}) , \quad 4d : a(\Phi^{(p)}) = -2p a(V^{\mathcal{N}=4}) . \quad (4.5)$$

For $p = 1$ we get the singleton multiplet values (taking into account the relation (2.4) between the boundary conformal field a-coefficient and the AdS field a^+ -coefficient) while for $p = 2$ we find the -26 and -4 coefficients in (4.2) and (4.3).¹⁹

case [6] and also in the strong-coupling limit of (2,0) theory as predicted by the AdS/CFT [44]. Thus in the (2,0) case there should be just one overall c-anomaly coefficient in (1.1). Then the same should be true also for the (2,0) CSG (which is equivalent to its 1-loop renormalizability). Let us stress that in the case of lower (1,0) supersymmetry this pattern will not apply – W_6 will no longer be universally proportional to \mathcal{W}_6 – one will need to consider two independent c_i coefficients. We find from the expressions in [6] that for the (1,0) scalar multiplet $S^{(1,0)}$ the c_i values are $(c_1, c_2, c_3) = (-\frac{1}{27}, -\frac{1}{540}, \frac{1}{180})$ and for the tensor multiplet $T^{(1,0)}$ they are $(-\frac{8}{27}, -\frac{11}{135}, \frac{1}{45})$ so one has a relation $c_1 - 2c_2 + 6c_3 = 0$ (see also [54, 55]).

¹⁷An independent confirmation of that would be to show the cancellation of axial anomalies as was done for the finite 4d system in [56].

¹⁸We thank K. Intriligator for this suggestion.

¹⁹Curiously, while this 26 coefficient has no apparent connection to critical dimension of bosonic string (apart from the remark made below eq. (4.4)) it originates from the same type of quadratic polynomial

The system of (2,0) conformal supergravity coupled to exactly 26 (2,0) tensor multiplets has also the following remarkable interpretation. Let us recall that starting with $n + 5$ (2,0) tensor multiplets coupled to (2,0) CSG background and spontaneously breaking the dilatation symmetry by imposing a quadratic constraint [20, 60] on $5 \times (n + 5)$ tensor multiplet scalars one ends up with a system of n tensor multiplets coupled [61, 62] to (2,0) 6d Poincaré supergravity. The remaining $5n$ scalars parametrize the coset $\frac{SO(5,n)}{SO(5) \times SO(n)}$.²⁰ Thus, starting with (2,0) CSG coupled to $26 = 5 + 21$ tensor multiplets and (i) spontaneously breaking superconformal symmetry and (ii) decoupling the higher-derivative terms by considering a low-energy limit, we end up with chiral (2,0) 6d Poincaré supergravity coupled to 21 (2,0) tensor multiplets. The latter theory is gravitational anomaly free (and also results upon compactification of type IIB supergravity on K3) [21, 22].

It may be possible to establish a connection of this fact to the a-anomaly cancellation in the (2,0) CSG + 26 tensor multiplet system if we assume that gravitational anomalies also cancel in that theory. As the chiral anomalies in the broken and unbroken phases should match, that would suggest (following a related discussion in the 4d case in section 2.2 of [65]) the cancellation of the gravitational anomaly also in the corresponding IR theory in the spontaneously broken phase, i.e. in the above (2,0) Poincaré supergravity coupled to 21 tensor multiplets.

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A Partition functions of 6d conformal fields on S^6

2-derivative vector $V_\mu^{(2)}$

Let us start with 2-derivative conformal 6d vector $V_\mu^{(2)}$ in (3.1). It is a special $s = 1$ case of a family of actions for conformal fields described by rank s symmetric tensor $\varphi_{\mu_1 \dots \mu_s}$

$6p^2 - 6p + 1$ as the ghost central charge of the bosonic string [58] (in the latter case p plays the role of the spin of the ghost system).

²⁰ Explicitly, one imposes $\eta^{IJ} \varphi_I^{ij} \varphi_{Jkl} = M^4 \eta_{kl}^{ij}$, where $\eta_{IJ} = \text{diag}(- - - - + \dots)$, $I, J = 1, \dots, 5 + n$, $i, j = 1, \dots, 4$ and M is mass scale parameter that determines the gravitational constant of the Poincaré supergravity. Here 5 tensor multiplets are chosen to be ghost-like to get the standard physical sign of the Poincaré supergravity action. The number 5 is also directly related to the presence of 5 rank 3 self-dual antisymmetric tensors in the (2,0) CSG spectrum in Table 2: they couple to the antisymmetric tensor field strength $H_{\mu\nu\lambda}$ of the tensor multiplets via $H_{\mu\nu\rho} T_{\mu\nu\rho}$ term and (together with associated conformal and the S-supersymmetry fixing) this effectively eliminates the ghost-like dynamics of the 5 tensor multiplets. This is a 6d counterpart of the construction of $\mathcal{N} = 4$ Poincaré supergravity coupled to n $\mathcal{N} = 4$ vector multiplets in 4d by starting with the system of $n + 6$ $\mathcal{N} = 4$ vector multiplets in $\mathcal{N} = 4$ conformal supergravity background. In this case the 6 ghost-like vectors couple to 6 self-dual tensors $T_{\mu\nu}$ of $\mathcal{N} = 4$ CSG via $F_{\mu\nu} T_{\mu\nu}$, etc. (for detailed discussions with some applications see [63–65]).

which in $d = 6$ have no gauge invariance. The corresponding Weyl invariant action on curved background was found in [16] (see also [66]). Here we shall use this action to compute the a-anomaly corresponding to $V_\mu^{(2)}$ by direct 6d method (i.e. from partition function on S^6 , see (2.2)) and check that it matches the value in Table 1 found from (2.5).

The Weyl invariant action for $V_\mu^{(2)} \equiv \varphi_\mu$ is

$$S(V^{(2)}) = \frac{1}{2} \int d^6x \sqrt{g} \left[\nabla^\lambda \varphi^\mu \nabla_\lambda \varphi_\mu - \frac{3}{2} (\nabla^\lambda \varphi_\lambda)^2 + \frac{1}{2} R_{\mu\nu} \varphi^\mu \varphi^\nu + \frac{3}{20} R \varphi^\mu \varphi_\mu \right]. \quad (\text{A.1})$$

To diagonalize the kinetic operator, we split φ_μ into transverse and longitudinal parts

$$\varphi_\mu = \varphi_{\mu\perp} + \nabla_\mu \sigma, \quad \nabla^\mu \varphi_{\mu\perp} = 0. \quad (\text{A.2})$$

Specializing to unit-radius S^6 (with $R_{\mu\nu} = \frac{R}{6} g_{\mu\nu}$, $R = 30$), we get

$$S = \frac{1}{2} \int d^6x \sqrt{g} \left[\varphi_\perp^\mu (-\nabla^2 + 7) \varphi_{\mu\perp} - \frac{1}{3} \sigma (-\nabla^2 + 6) \nabla^2 \sigma \right]. \quad (\text{A.3})$$

Taking into account the Jacobian for the change of variables in (A.2), we obtain for the partition function

$$Z(V^{(2)}) = \left[\frac{\det \hat{\Delta}_0(0)}{\det \hat{\Delta}_{1\perp}(7) \det \hat{\Delta}_0(6) \det \hat{\Delta}_0(0)} \right]^{1/2} = \left[\frac{1}{\det \hat{\Delta}_{1\perp}(7) \det \hat{\Delta}_0(6)} \right]^{1/2}, \quad (\text{A.4})$$

where $\hat{\Delta}_{1\perp}$ is a special case of the operator $\hat{\Delta}_{s\perp}(M^2) = (-\nabla^2 + M^2)_{s\perp}$ acting on transverse traceless rank s tensors (with $\hat{\Delta}_0(M^2) = -\nabla^2 + M^2$ being scalar Laplacian). As a result, we get from (2.2)

$$a(V^{(2)}) = -\frac{1}{96} \left(B_6[\hat{\Delta}_{1\perp}(7)] + B_6[\hat{\Delta}_0(6)] \right), \quad (\text{A.5})$$

where B_6 are the corresponding Seeley coefficients. The general expression for $B_6[\hat{\Delta}_{s\perp}(M^2)]$ (applicable also to $s = 0$ case) was found in [7]

$$\begin{aligned} B_6[\hat{\Delta}_{s\perp}(M^2)] &= \frac{(s+1)(s+2)(2s+3)}{453600} \left[-210 M^6 - 315 M^4 (s^2 + s - 10) \right. \\ &\quad \left. + 630 M^2 (s+6)(s^2 + 2s - 4) \right. \\ &\quad \left. + 22780 - 18774s - 19488s^2 - 4515s^3 - 315s^4 \right]. \end{aligned} \quad (\text{A.6})$$

Computing the two terms in (A.5), we finally get

$$a(V^{(2)}) = -\frac{1}{96} \left(\frac{67}{3780} + \frac{1}{756} \right) = -\frac{1}{7!}. \quad (\text{A.7})$$

This is in agreement with the $V^{(2)}$ entry in Table 1 derived from (2.5), i.e. using AdS₇-based method.

4-derivative vector $V_\mu^{(4)}$

The 4-derivative $V_\mu^{(4)} \equiv A_\mu$ vector field in (3.1) with gauge-invariant action is the $s = 1$ member of the conformal higher spin (CHS) family in $d = 6$ [7]. Considering generic

curved background and assuming that under Weyl transformations $g'_{\mu\nu} = e^{2\rho} g_{\mu\nu}$ the gauge field is not transforming $A'_\mu = A_\mu$ we find that the corresponding Weyl-invariant action is (here $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$)²¹

$$S(V^{(4)}) = \int d^6x \sqrt{g} \left[\nabla_\lambda F^\lambda_\mu \nabla^\mu F^{\nu\lambda} - (R_{\mu\nu} - \frac{1}{5} R g_{\mu\nu}) F^{\mu\lambda} F^\nu_\lambda \right]. \quad (\text{A.8})$$

Integrating by parts and using $\partial_{[\mu} F_{\nu\lambda]} = 0$ this action can be written also as

$$S(V^{(4)}) = \frac{1}{2} \int d^6x \sqrt{g} F^{\mu\nu} \left(-\nabla^2 F_{\mu\nu} + \frac{2}{5} R F_{\mu\nu} - 2 R_{\mu\kappa\nu\lambda} F^{\kappa\lambda} \right). \quad (\text{A.9})$$

Note that (A.8) may be viewed as a 6d Weyl-invariant action for a zero Weyl weight 2-form field $B_{\mu\nu} = F_{\mu\nu}$ only assuming it satisfies the Bianchi constraint, i.e. is longitudinal. An alternative Weyl invariant action that depends only on the transverse part of B is the familiar one $\int d^6x \sqrt{g} H^{\mu\nu\lambda} H_{\mu\nu\lambda}$ where $H = dB$ (cf. [67]).

Specifying to S^6 with $R_{\mu\kappa\nu\lambda} = g_{\mu\nu} g_{\kappa\lambda} - g_{\mu\lambda} g_{\kappa\nu}$ we get from either (A.8) or (A.9) with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ (cf. (A.3))

$$S(V^{(4)}) = \int d^6x \sqrt{g} A_\perp^\mu (-\nabla^2 + 7)(-\nabla^2 + 5) A_{\mu\perp}. \quad (\text{A.10})$$

Taking into account the Jacobian of the change of variables $A_\mu \rightarrow A_{\mu\perp} + \nabla_\mu \sigma$ one finds then the following partition function (note that $\nabla_\lambda F^\lambda_\mu = (-\nabla^2 + 5) A_{\mu\perp}$) (cf. (A.4))

$$Z(V^{(4)}) = \left[\frac{\det \hat{\Delta}_0(0)}{\det \hat{\Delta}_{1\perp}(7) \det \hat{\Delta}_{1\perp}(5)} \right]^{1/2}. \quad (\text{A.11})$$

This agrees with the $s = 1$ case of the general expression for the 6d partion function of the CHS field in [7].

Using (A.6) that gives (cf. (A.7))

$$a(V^{(4)}) = -\frac{1}{96} \left(\frac{67}{3780} - \frac{1403}{3780} - \frac{1139}{3780} \right) = \frac{1}{7!} \frac{275}{8}, \quad (\text{A.12})$$

in agreement with the $V^{(4)}$ value in Table 1. Here the middle entry is that of the operator $\hat{\Delta}_{1\perp}(5)$. The latter appears also in the S^6 partition function of the standard (non-conformal) 6d Maxwell vector (here $-g_{\mu\nu} \nabla^2 + R_{\mu\nu} \rightarrow g_{\mu\nu} (-\nabla^2 + 5)$, cf. (A.4), (A.11))

$$Z(6d \text{ Maxwell}) = \left[\frac{\det \hat{\Delta}_0(0)}{\det \hat{\Delta}_{1\perp}(5)} \right]^{1/2}. \quad (\text{A.13})$$

The difference between (A.13) and (A.11) is thus in the contribution of the operator $\hat{\Delta}_{1\perp}(7)$ which is the same as the transverse part of the $V^{(2)}$ operator in (A.3). For (A.13), the

²¹We use that for $J = \frac{1}{2(d-1)} R$, $K_{\mu\nu} = \frac{1}{d-2} (R_{\mu\nu} - J g_{\mu\nu})$ in d dimensions one has $\delta J = -2\rho J - \nabla^2 \rho$, $\delta K_{\mu\nu} = -\nabla_\mu \nabla_\nu \rho$. One can also add with an arbitrary coefficient a term with the Weyl tensor $\int d^6x \sqrt{g} C_{\mu\kappa\nu\lambda} F^{\mu\nu} F^{\kappa\lambda}$ which is separately Weyl-invariant.

coefficient of logarithmically divergent part of $\log Z$ in (2.2) (which here does not have the standard conformal anomaly interpretation so we will call it "a") is²²

$$''a''(6d \text{ Maxwell}) = -\frac{1}{96} \left(-\frac{1403}{3780} - \frac{1139}{3780} \right) = \frac{1}{7!} \frac{1271}{36}. \quad (\text{A.14})$$

In 6 dimensions a gauge-invariant vector A_μ is dual to rank 3 antisymmetric tensor $C_{\mu\nu\lambda}$, e.g., the Maxwell action $F_{\mu\nu}F^{\mu\nu}$ is dual to $H_{\mu\nu\lambda\rho}H^{\mu\nu\lambda\rho}$ where $H_4 = dC_3$. Starting with flat space case and considering instead the conformally invariant 4-derivative $V^{(4)}$ Lagrangian $L = F_{\mu\nu}\partial^2 F^{\mu\nu}$ we get as its dual a non-local $\tilde{L} = -\frac{1}{4}H^{\mu\nu\lambda\rho}\partial^{-2}H_{\mu\nu\lambda\rho} = C_\perp^{\mu\nu\lambda}C_{\mu\nu\lambda\perp}$, where $\partial^\mu C_{\mu\nu\lambda\perp} = 0$. The corresponding action is conformal and may be viewed as the analog of the conformal Schwinger action $\int d^2x (A_\mu)_\perp^2$ for a vector in 2 dimensions.

Dualizing the curved space action (A.8) or (A.9) by first adding $\epsilon^{\mu\nu\lambda\rho\kappa\delta}F_{\mu\nu}H_{\lambda\rho\kappa\delta}$ and then integrating out $F_{\mu\nu}$ one ends up with a non-local action for $H_{\lambda\rho\kappa\delta}$. For example, for S^6 background when the integrand in (A.9) becomes $F^{\mu\nu}(-\nabla^2 + 10)F_{\mu\nu}$ we get²³

$$\tilde{L} = -\frac{1}{4}H^{\mu\nu\lambda\rho}(-\nabla^2 + 10)_\perp^{-1}H_{\mu\nu\lambda\rho} = C_\perp^{\mu\nu\lambda}(-\nabla^2 + 9)_\perp(-\nabla^2 + 11)_\perp^{-1}C_{\mu\nu\lambda\perp}. \quad (\text{A.15})$$

To show this one notes that the Lagrangian may be written as $-C^{\mu\kappa\lambda}\nabla^\rho\frac{1}{-\nabla^2+10}\nabla_\rho C_{\mu\kappa\lambda} - 3C^{\mu\kappa\lambda}\nabla^\rho\frac{1}{-\nabla^2+10}\nabla_\lambda C_{\mu\kappa\rho}$. Then one may expand $\frac{1}{-\nabla^2+10} = \frac{1}{10}(1 + \frac{1}{10}\nabla^2 + \frac{1}{100}\nabla^4 + \dots)$ and commute covariant derivatives using that only the transverse part of C_3 should contribute.²⁴

The dual C_3 theory is thus defined by the path integral $Z(C) = \int [dC_3] G e^{-\tilde{S}(C_3)}$ with the gauge-invariant but non-local action (A.15) and $G = [\det \hat{\Delta}_F(10)]^{-1/2}$ (with $\hat{\Delta}_F(M^2) \equiv -\nabla^2 + M^2$ acting on a rank 2 antisymmetric tensor) being the factor coming from integrating out F in the 1st order action $F_2\Delta_F(10)F_2 + \epsilon_6 F_2 H_4$. Equivalently, we may write $G = [\det \hat{\Delta}_{F\perp}(10) \det \hat{\Delta}_{1\perp}(7)]^{-1/2}$. This unfamiliar factor (irrelevant at the classical level) is important for quantum equivalence of the two dual theories. One can show that the partition function resulting from integration over C_3 is indeed equivalent to (A.11). Splitting $C_3 = C_{3\perp} + dB_2$ where B_2 is a 2-form that can be also decomposed as $B_2 = B_{2\perp} + dA$, etc., we get the Jacobian which is the inverse of the standard 2-form partition function in (A.16), i.e. $J = 1/Z(B) = [\det \hat{\Delta}_{B\perp}(8) \det \hat{\Delta}_0(0)]^{1/2} [\det \hat{\Delta}_{1\perp}(5)]^{-1/2}$. Integrating out $C_{3\perp}$ gives $[\det \hat{\Delta}_{C\perp}(9)]^{-1/2} [\hat{\Delta}_{C\perp}(11)]^{1/2}$. Now using the relation between S^6 determinants of 2-form and 3-form operators [68] $\det(-\nabla^2 + m^2)_{B\perp} = \det(-\nabla^2 + m^2 + 1)_{C\perp}$ one finds that $Z(C)$ is equivalent to $Z(V^{(4)})$ in (A.11).

A potential interest in considering the action for C_3 like $C_{3\perp}C_{3\perp} + \dots$ as a dual alternative to the conformal 4-derivative vector theory is that a similar term appeared in the context of (1,0) superconformal models with non-abelian tensor fields [18, 69]. For example,

²² Then for (1,0) Maxwell multiplet containing in addition two MW fermions ψ in Table 1 we get $''a''((1,0) \text{ Maxwell}) = \frac{1}{7!}(\frac{1271}{36} - \frac{191}{144}) = \frac{1}{7!}\frac{1631}{48}$.

²³ Note that $\nabla^\mu H_{\mu\nu\lambda\rho} = (-\nabla^2 + 9)C_{\nu\lambda\rho\perp}$. In general, for Hodge-deRham operator acting on a p -form in d dimensions one has (see, e.g., [68]): $\hat{\Delta} = -\nabla^2 + p(d-p)$. To integrate over the 2-form F one needs to split $F = F_\perp + dA$ which brings in the Jacobian $[\det \hat{\Delta}_{1\perp}(5)]^{-1/2}$ and to notice that C_3 couples only to F_\perp .

²⁴ Explicitly, $-C_\perp^{\mu\kappa\lambda}\nabla^\rho(\nabla^2)^n\nabla_\rho C_{\mu\kappa\lambda\perp} - 3C_\perp^{\mu\kappa\lambda}\nabla^\rho(\nabla^2)^n\nabla_\lambda C_{\mu\kappa\rho\perp} = C_\perp^{\mu\kappa\lambda\perp}P_n(\nabla^2)C_{\mu\kappa\lambda\perp}$ where P_n is a polynomial in ∇^2 which can be shown to be $P_n = \frac{1}{10^{n+1}}(9 - \nabla^2)(\nabla^2 - 1)^n$.

a conformal model $L = (C_3 + dB_2)^2 + \phi(B_2 + dA)^2 + \dots$ in the phase where ϕ has trivial expectation value will have its conformal anomaly determined by the first term which has an effective gauge invariance, i.e. by $(C_{3\perp})^2 + (dB'_2)^2$. The contribution of $(C_{3\perp})^2$ to the conformal anomaly will be coming just from the measure or ghost factors and will thus be like that of a non-unitary theory.

Antisymmetric tensor $T_{\mu\nu}$

The S^6 partition function for antisymmetric tensor $T_{\mu\nu}$ with the standard gauge-invariant Lagrangian $H_{\mu\nu\lambda}H^{\mu\nu\lambda}$, $H = dT$ (we relax self-duality condition here) can be found, *e.g.*, from the general curved space expression in [70, 6]

$$Z(T) = \left[\frac{(\det \hat{\Delta}_{1\perp}(5))^2}{\det \hat{\Delta}_T(8) (\det \hat{\Delta}_0(0))^3} \right]^{1/2} = \left[\frac{\det \hat{\Delta}_{1\perp}(5)}{\det \hat{\Delta}_{T\perp}(8) \det \hat{\Delta}_0(0)} \right]^{1/2}, \quad (\text{A.16})$$

where $\hat{\Delta}_T T_{\mu\nu} = (-\nabla^2 + M^2)T_{\mu\nu}$ and $\hat{\Delta}_{T\perp}$ acts on transverse antisymmetric 2-tensors.²⁵ Note that the partition function of T -field with the contribution from its action $\det \hat{\Delta}_{T\perp}(8)$ omitted is just the inverse of the 6d Maxwell partition function (A.13).

Relation to conformal group representations

Comparing the structure of the above 6d partition functions on S^6 to the conformal group representation content of the respective fields in Table 1 we conclude that the correspondence is not immediately straightforward. In general, given a field in some representation $(\Delta \equiv \Delta_+; h_1, h_2, h_3)$ it will correspond to $Z = D^{-1/2}$ where D is some product of determinants of 2nd order Laplacians on S^6 . This is, indeed, expected for higher derivative conformal scalar (GJMS) operators. For totally symmetric traceless conformal tensors the general rule appears to be

$$(\Delta; s, 0, 0) \longrightarrow \prod_{k=1}^{\Delta-3} \prod_{s'=0}^s \det \hat{\Delta}_{s'\perp}(M_{s',k-2}^2), \quad M_{s',k-2}^2 = 6 + s' - k(k-1), \quad (\text{A.17})$$

where $M_{k,n}^2 = k - (n-1)(n+d-2)$ or $k - (s-1)(s+4)$ in $d=6$. One can check that the corresponding a-coefficient computed via 7d relation (2.5) is the same as found using 6d approach based on (A.6), *i.e.*

$$a(\Delta; s, 0, 0) = -\frac{1}{96} \sum_{k=1}^{\Delta-3} \sum_{s'=0}^s B_6[\hat{\Delta}_{s'\perp}(M_{s',k-2}^2)]. \quad (\text{A.18})$$

²⁵Here the first equality is found in the usual covariant Feynman gauge while to derive the second one we should set $T_{\mu\nu} = T_{\mu\nu\perp} + B_{\mu\nu}$, $B_{\mu\nu} = \nabla_\mu Q_\nu - \nabla_\nu Q_\mu$, and account for the Jacobian $J_1 = [\det \hat{\Delta}_{1\perp}(5)]^{1/2}$ of this change of variables. Note that without gauge fixing the classical Lagrangian is $H_{\mu\nu\lambda}H^{\mu\nu\lambda} \sim T_\perp \hat{\Delta}_{T\perp}(8) T_\perp$ and the remaining factors in the second form of (A.16) come from the Jacobian J_1 and the fact that one is to divide out the full gauge group volume $[dQ_\mu] = dQ_{\mu\perp} d\alpha [\det \hat{\Delta}_0(0)]^{1/2}$ (here we set $Q_\mu = Q_{\mu\perp} + \partial_\mu \alpha$). Note also that on a general curved background $(\hat{\Delta}_T)_{\mu\nu}^{\lambda\rho} = -(\nabla^2)_{\mu\nu}^{\lambda\rho} + 2R_{[\mu}^{\lambda} \delta_{\nu]}^{\rho]} - R_{\mu\nu}^{\lambda\rho}$ so that for a unit-radius S^6 where $R_{\lambda\rho\mu\nu} = g_{\lambda\mu}g_{\rho\nu} - g_{\lambda\nu}g_{\rho\mu}$ we thus get $(\hat{\Delta}_T)_{\mu\nu}^{\lambda\rho} = (-\nabla^2 + 8)_{\mu\nu}^{\lambda\rho}$. The action before gauge fixing depends only on $T_{\mu\nu\perp}$ while the standard gauge-fixing term $(\nabla^\mu T_{\mu\nu})^2$ gives $(\nabla^\mu C_{\mu\nu})^2$ or $[(-\nabla^2 + 5)C_{\mu\perp}]^2$.

The case of $s = 0$ corresponds to scalar GJMS operators (see, e.g., [71–73]) where (A.17) reproduces the known relations, e.g.,

$$(4, 0, 0, 0) \longrightarrow \det \hat{\Delta}_0(6), \quad (6, 0, 0, 0) \longrightarrow \det \hat{\Delta}_0(6) \det \hat{\Delta}_0(4) \det \hat{\Delta}_0(0). \quad (\text{A.19})$$

For $s = 1$ some relevant special cases are

$$\begin{aligned} (4; 1, 0, 0) &\longrightarrow \det \hat{\Delta}_{1\perp}(7) \det \hat{\Delta}_0(6), \\ (5; 1, 0, 0) &\longrightarrow \det \hat{\Delta}_{1\perp}(7) \det \hat{\Delta}_{1\perp}(5) \det \hat{\Delta}_0(6) \det \hat{\Delta}_0(4). \end{aligned} \quad (\text{A.20})$$

Since, according to Table 1, the vector $V^{(2)}$ corresponds to $(4; 1, 0, 0)$ and $V^{(4)}$ to $(5; 1, 0, 0) - (6; 0, 0, 0)$ representations, we see that (A.20), (A.19) are indeed consistent with the partition functions in (A.4) and (A.11).

For $s = 2$ the important special case is the conformal graviton in Table 2. Here one finds from (A.17)

$$Z(h_{\mu\nu}) = Z[(6; 2, 0, 0) - (7; 1, 0, 0)] = \left[\frac{\det \hat{\Delta}_{1\perp}(-5) \det \hat{\Delta}_0(-6)}{\det \hat{\Delta}_{2\perp}(8) \det \hat{\Delta}_{2\perp}(6) \det \hat{\Delta}_{2\perp}(2)} \right]^{1/2}, \quad (\text{A.21})$$

which is in agreement with the $s = 2$ CHS expression in [7] (see also [74]).

Similarly, in the antisymmetric tensor case $T = (4; 1, 1, 0) - (5; 1, 0, 0) + (6; 0, 0, 0)$ (cf. Table 1) eq.(A.16) together with (A.19), (A.20) imply the following correspondence

$$(4; 1, 1, 0) \longrightarrow \det \hat{\Delta}_{T\perp}(8) \det \hat{\Delta}_{1\perp}(7). \quad (\text{A.22})$$

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